

Inertia Tensor for MORVEL Tectonic Plates^{*}

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Abstract: The NNR (No-Net-Rotation)-MORVEL (Mid-Ocean Ridge VELOCITY) 56 is a set of angular velocities describing the motions of 56 plates relative to a No-Net-Rotation reference frame. These plates can be adjusted in terms of non-overlapping polygonal regions, separated by plate boundaries on a unit sphere. During the calculation on the kinematic parameters for these 56 plates in a NNR reference frame using the International Terrestrial Reference Frame (ITRF) velocity field, the geometric parameters of tectonic plates play a significant role in establishing an absolute plate motion model based on space geodesy results. The computational method for these geometric parameters implemented as a FORTRAN90 program is described in this paper, allowing an evaluation of the area and the inertia tensor of a polygonal region on a unit sphere. This program is mainly built on a triangulation algorithm and the adaptive Simpson's double integral method for spherical polygons, which produces highly reliable results for all 56 modern plates.

Key words: Tectonic plate; Spherical polygon; Inertia tensor; NNR-MORVEL56

1 Introduction

Most of Earth's major features can be understood from the interactions between tectonic plates, which move independently, separating from, colliding with, and sliding against one another. Until the middle 1960s an unifying theory was developed to explain Earth's dynamics^[1]. Several decades after the inception of the theory on plate tectonics, the plate dynamic models constructed using the geological and geophysical data have been dominant, until long time-span geodetic observations were gathered to estimate contemporary plate kinematic parameters^[2-5]. As one of the most representative geological plate motion models, NUVEL-1A is one of the mainstream models regarding the plate dynamics and kinematics. With the setting up of the enhanced amount and quality of the geologic or geodetic data during the last few years, the MORVEL refined the precision and accuracy of the geometric and kinematic parameters for 56 plates that are partly taken from an updated digital model of plate boundaries by Bird^[6]. Relative to the NUVEL-1A, the MORVEL incorporates more than twice as many plates and covers more of Earth's surface, and nearly all the NUVEL-1A angular velocities differ significantly from its MORVEL counterparts^[7].

To derive an absolute motion model in a NNR reference frame, however, the inertia tensors are always considered as indispensable attribute of all these plates. Despite various established methods for calculating plate inertia tensors corresponding to the NUVEL-1A model presented in many papers^[8-9], however it is necessary to recalculate a new set for the NNR-MORVEL56 model^[10], given the considerable discrepancy between the NUVEL-1A and the MORVEL.

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When a polygon on the unit sphere is employed for the representation of a tectonic plate, a simplified analysis of the plate inertia tensor can be performed through a numerical method, which is carried out over all 56 plates. The method for calculating all 9 components of the inertia tensor is illustrated in this paper and this method requires the precise knowledge of the plate boundaries. The boundary file contains a 2-column sequence of the latitude-longitude plate boundary coordinates that fully enclose the plate in the counterclockwise direction.

In the first section, we introduce some concepts regarding the no-net-rotation conditions and indicate the calculation of the Euler vector in an absolute motion model. The following section describes the detailed mathematical models to estimate the area and the inertia tensor of the spherical polygons. The last section of this paper is dedicated to manifest the results for 56 modern plates and the appendix gives the original FORTRAN90 program for obtaining the aforementioned results.

2 Net Lithosphere Rotation

A no-net-rotation model for the lithosphere assumes that the integral of $v \times r$ over the Earth's surface equals zero^[11], i. e.

$$\int_{\text{Earth}} v \times r dS = 0, \quad (1)$$

where, r is the radial vector of the surface element on a unit sphere, and v corresponds to the horizontal velocity at that position. The angular velocity of net rotation ω_{net} was computed as the total angular momentum of all plates divided by the moment of inertia of the entire lithosphere, using the equation^[12-13]

$$\omega_{\text{net}} = \frac{3}{8\pi} \int_{\text{Earth}} v \times r dS. \quad (2)$$

Then it is convenient to convert the Eq.(2) into the following form^[8]:

$$\omega_{\text{net}} = \frac{3}{8\pi} \sum_{i=1}^n Q_i \omega_i, \quad (3)$$

where ω_i is the Euler vector describing the motion of plate i relative to an inertial reference frame, such as ITRF2008, and Q_i is the inertia tensor of plate i , where i goes from 1 to n . The angular velocities of the plates relative to the NNR reference frame were then found by vector subtraction, namely,

$$\omega_i^{\text{NNR}} = \omega_i - \omega_{\text{net}}. \quad (4)$$

For those tectonic plates where angular velocities are not available in the geodetic model such as the ITRF2000-PMM, due to a lack of sufficient data, Altamimi^[14] tested four cases to perfect the incomplete geodetic model. The fourth case described a method for estimating the missing angular velocity. Here we employ it in this paper by using a simple equation written as:

$$\omega_i^{\text{ITRF}} = \omega_{ij}^{\text{MORVEL}} + \omega_j^{\text{ITRF}}, \quad (5)$$

where ω_i^{ITRF} is the undetermined rotation vector of plate i in the ITRF model, and $\omega_{ij}^{\text{MORVEL}}$ is the MORVEL rotation vector for plate i relative to plate j , which is adjacent to the missing plate i .

3 Area and inertia tensor of plates

3.1 Evaluation of the plate area

The spherical polygons are defined by great circle arcs connecting points on the sphere, the positions of which are given by latitudes and longitudes. In fact, an algorithm for determining the area of a spherical polygon of arbitrary shape has been presented by Bevis and Cambareri^[15], where the kernel idea is to compute the interior angle at each vertex of the spherical polygon. In this paper, however, we employ a somewhat similar method to that of Miller^[16], trying to determine the area of spherical polygon by summing the signed

areas of component triangles.

For a spherical polygon of n sides, the spherical excess E is generalized as

$$E = \sum_{i=1}^n \alpha_i - ((n-2) \times 180^\circ), \quad (6)$$

where $\alpha_i (i=1 \cdots n)$ are the interior angles of the polygon. Considering a spherical polygon ABCD as shown in Fig. 1(a), the north-pole combined with any two adjacent vertices of the polygon can constitute a spherical triangle, such as NAB. The two sides of the triangle are known from the latitudes of their vertices, i. e. $an = \pi/2 - \text{latitude}(A)$ and $bn = \pi/2 - \text{latitude}(B)$. Taking the previously obtained two sides and the included angle specified by the difference between longitudes of A and B , the opposite side ab can be calculated via the formula:

$$ab = 2\sin^{-1} \sqrt{\text{hav}(bn - an) + \sin bn \sin an \text{hav} \angle N}, \quad (7)$$

where the haversine function is defined as $\text{hav} x = (1 - \cos x)/2$ with x in radians. Having obtained all three sides of the spherical polygon, we can use the formula to obtain its excess:

$$E = 4 \tan^{-1} \sqrt{\tan \frac{s}{2} + \tan \frac{s - an}{2} + \tan \frac{s - bn}{2} + \tan \frac{s - ab}{2}}, \quad (8)$$

where $s = \frac{an + bn + ab}{2}$.

To find the area of a spherical polygon, first, one may use the successive vertices in pair to form a spherical triangle. Each spherical triangle employs the north pole as a common vertex to make the calculations convenient. When calculating the areas of the individual triangles, we adopt a convention that the sign of the triangle area (which has a same value as the spherical excess for a polygon on a unit sphere) is identical to the sign of the difference between the longitudes of a pair of adjacent vertices. If the longitude of the first vertex is less than the second one, then the sign of this triangle area is defined as a positive value for a set of points arranged in the anticlockwise way and vice versa. Therefore the area of the spherical polygon is regarded as the absolute value of the sum of the signed spherical excesses for each of the spherical triangles. Taking the facility of the calculation for the upcoming inertia tensor, a provision is crucial that the vertex point traversing the polygon prefersto be enumerated in the counterclockwise direction.

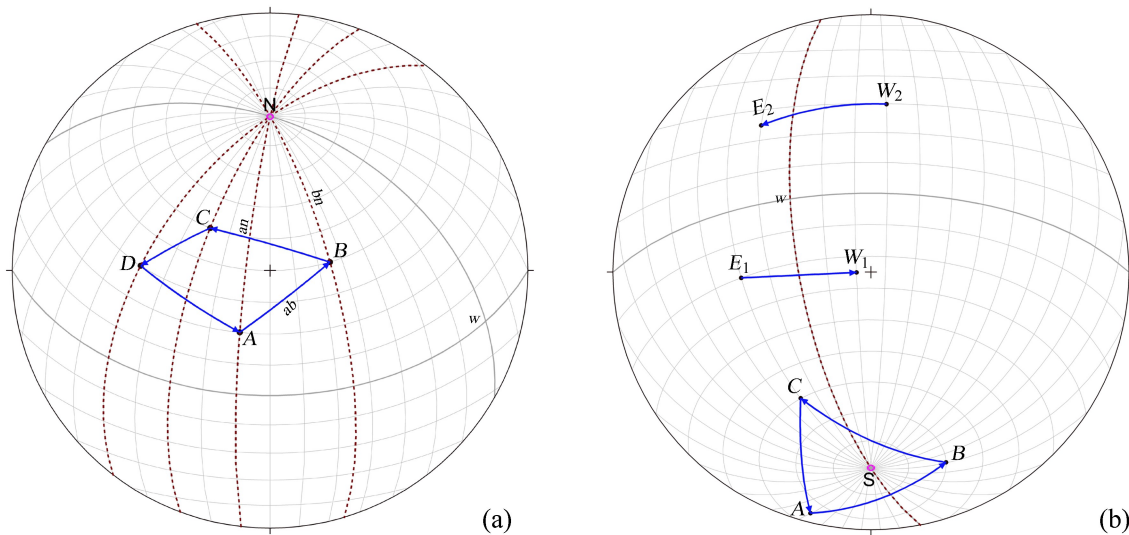


Fig. 1 Spherical polygon and triangles illustrating calculation method for area and inertia tensor discussed in text.

(a) N triangles constructed from counterclockwise spherical polygon of n sides;

(b) Spherical triangle encompassing South Pole and the sides of polygon traversing the 180th meridian

One should note the following two special cases when estimating the geometric parameters of the spherical polygons. The first case is as follows: if the sides of the polygon cross over the International Date Line from

point E_1 to W_1 , as illustrated in the Fig. 1(b), a 2π has to be added to the longitude of W_1 . The same applies to a similar situation when a vertex leave W_2 for E_2 , a 2π has to be subtracted from E_2 in order to promise the continuity of the calculation. The second case occurs when a spherical polygon has an area larger than that of the hemisphere. According to a implicit assumption that the area of any polygon is less than 2π , for those extensive polygons, such as the triangle ABC surrounding the north pole N , the exact area of ABC is the complement of ABC including the north pole, i. e. $Exactarea = Area_{ABC}^S = 4\pi - Area_{ABC}^N$. The original FORTRAN90 program in the appendix has taken into account these two singular cases.

3.2 Estimation of plate inertia tensor

The components of the symmetric inertia tensor Q can be calculated for a region P using the following formula:

$$Q_{\mu\nu} = \int_P (\delta_{\mu\nu} - x_\mu x_\nu) dA, \quad (9)$$

where x_μ ($\mu = 1, 2, 3$) are the Cartesian coordinates, $\delta_{\mu\nu}$ ($\mu, \nu = 1, 2, 3$) are the elements of the identity matrix, and the integration is carried out over the surface of a plate P . These inertia tensors are based on the hypothesis that the surface density of the plate is unit one, and entirely describe the plate geometry. For instance, the plate area A is easily calculated by taking the trace of Q :

$$Tr(Q) = \sum_\mu \int_P (1 - x_\mu^2) dA = 2A. \quad (10)$$

This implies invariance of the trace under coordinate rotations and the sum of the diagonal components is always double the area of the polygon. Generally speaking, non-diagonal components indicate the asymmetry of the polygon with respect to the Cartesian axes, and all of the diagonal components have a positive value, which is useful, together with the Eq.(10), as the verification test for the calculation results.

In this paper we propose a somewhat different method from Schettino^[17] for constructing the spherical triangle. In fact, we will see that the integral at the right-hand side of (Eq.(9)) is easily calculated for spherical triangles. The components of the total tensor are therefore given by:

$$\begin{aligned} Q_{\mu\nu} &= \sum_{i=1}^n \int_{T_i} (\delta_{\mu\nu} - x_\mu x_\nu) dA \\ &= \delta_{\mu\nu} A - \sum_{i=1}^n \int_{T_i} x_\mu x_\nu dA \end{aligned}, \quad (11)$$

where A is the total polygon area, which has been illustrated in the last section. Let a point on the sphere be given in spherical coordinates (θ, λ) , where θ is the latitude and λ is the longitude, so that its Cartesian coordinates are given by

$$x_1 = \cos\theta\cos\lambda \quad x_2 = \cos\theta\sin\lambda \quad x_3 = \sin\theta, \quad (12)$$

then the area element dA at this position is equal to $\cos\theta d\lambda d\theta$. The components of the inertia tensor for a triangle NAB are therefore written, in spherical coordinates, as:

$$Q_{\mu\nu} = \delta_{\mu\nu} A - \int_{\lambda_1}^{\lambda_2} d\lambda \int_{\theta(\lambda)}^{\pi/2} f_{\mu\nu}(\theta, \lambda) d\theta, \quad (13)$$

where λ_1, λ_2 are the longitudes of vertices A, B and the function f is given by

$$f(\theta, \lambda) = \begin{bmatrix} f_{11}(\theta, \lambda) & f_{12}(\theta, \lambda) & f_{13}(\theta, \lambda) \\ f_{21}(\theta, \lambda) & f_{22}(\theta, \lambda) & f_{23}(\theta, \lambda) \\ f_{31}(\theta, \lambda) & f_{32}(\theta, \lambda) & f_{33}(\theta, \lambda) \end{bmatrix}. \quad (14)$$

Next, as a result of the symmetry of the inertia tensor, the 6 independent components of f are expressed in the following way:

$$f_{11}(\theta, \lambda) = \cos^3\theta\cos^2\lambda \quad (15)$$

$$f_{12}(\theta, \lambda) = \cos^3\theta\cos\lambda\sin\lambda \quad (16)$$

$$f_{13}(\theta, \lambda) = \cos^2\theta\sin\theta\cos\lambda \quad (17)$$

$$f_{22}(\theta, \lambda) = \cos^3 \theta \sin^2 \lambda \quad (18)$$

$$f_{23}(\theta, \lambda) = \cos^2 \theta \sin \theta \sin \lambda \quad (19)$$

$$f_{33}(\theta, \lambda) = \sin^2 \theta \cos \theta. \quad (20)$$

The upper limit of the inner integral about the latitude is always set to $\pi/2$, because the North Pole is considered as the common vertex of each of the spherical triangles. In contrast, with the simple upper limit, the lower limit function $\theta(\lambda)$ can be obtained from a series of derivations, whose concrete form is formulated as

$$\theta(\lambda) = -\arctan\left(\frac{C_1 \cos \lambda + C_2 \sin \lambda}{C_3}\right), \quad (21)$$

where C_1 , C_2 , C_3 represent three constants. Once the integrated triangles are determined by one side of the polygon, such as AB , we can write their expression in the following form:

$$\begin{aligned} C_1 &= \cos \theta_1 \sin \lambda_1 \sin \theta_2 - \cos \theta_2 \sin \lambda_2 \sin \theta_1 \\ C_2 &= \cos \theta_2 \cos \lambda_2 \sin \theta_1 - \cos \theta_1 \cos \lambda_1 \sin \theta_2, \\ C_3 &= \cos \theta_1 \cos \theta_2 \sin(\lambda_2 - \lambda_1) \end{aligned} \quad (22)$$

where (θ_1, λ_1) , (θ_2, λ_2) are the latitude and the longitude of vertices A and B , respectively.

Unlike the process of the area estimation, the evaluation of the inertia tensor is associated with the integral order. Hence, a counterclockwise direction must be adopted in the procedure. All of the special situations have been considered in the Fortran program, including the 180th meridian case and the encompassing the South Pole case. In addition to the aforementioned two special cases in the area estimation, the principal moments in the tensor calculation need to be deducted from the whole inertia tensor of the spherical surface in order to acquire the exact moments, i. e. $Moment_{\text{Exact}}^S = 8\pi/3 - Moment_{\text{Principal}}^N$, when involving the polygon containing the south pole.

4 Results and analysis

The NNR-NUVEL56 model contains 56 tectonic plates around the earth, and the software OSXStereonet developed by Cardozo and Allmendinger^[18] was applied to plot the global plate distribution map, as illustrated in Fig. 2. Utilizing the Fortran program, we estimated geometry parameters of all 56 modern plates with the accuracy the inertia tensor better than 10^{-6} . Table 1 lists the area and the inertia tensor of all MORVEL plates, which provide essential material for calculating plate kinematic parameters in the NNR reference frame. The sum of the area of all plates equals 12.566340 steradian, which is slightly less than the surface area of the whole unit sphere, namely 4π (12.566371) with the relative error $\eta = 0.00023\%$. Our results indicate that the relative errors for the six components of the total tensor are 0.00017%, 0.00029%, 0.00026%, 0.0010%, 0.00016%, and 0.00010% respectively. It is shown that the inertia tensor of the entire spherical surface is $8\pi/3E$, where E takes the identity matrix. The discrepancy probably arises from either the imperfection of plates over the entire sphere or the unavoidable rounding errors in floating point arithmetic.

5 Conclusion

The method for computing the areas and inertial tensors of tectonic plates has been presented. This method is based upon the triangulation algorithm and the adaptive Simpson's double integral procedure, which can be applied to the spherical polygons representing such tectonic plates. Results for the NNR-MORVEL56 tectonic plates show that highly reliable data can be produced, as long as starting from the precise definition of the plate boundaries. In addition, a FORTRAN90 program has been attached to the end of this paper, which is expected to be valuable to the future studies of the kinematics and dynamics associated to the motions of tectonic plates.

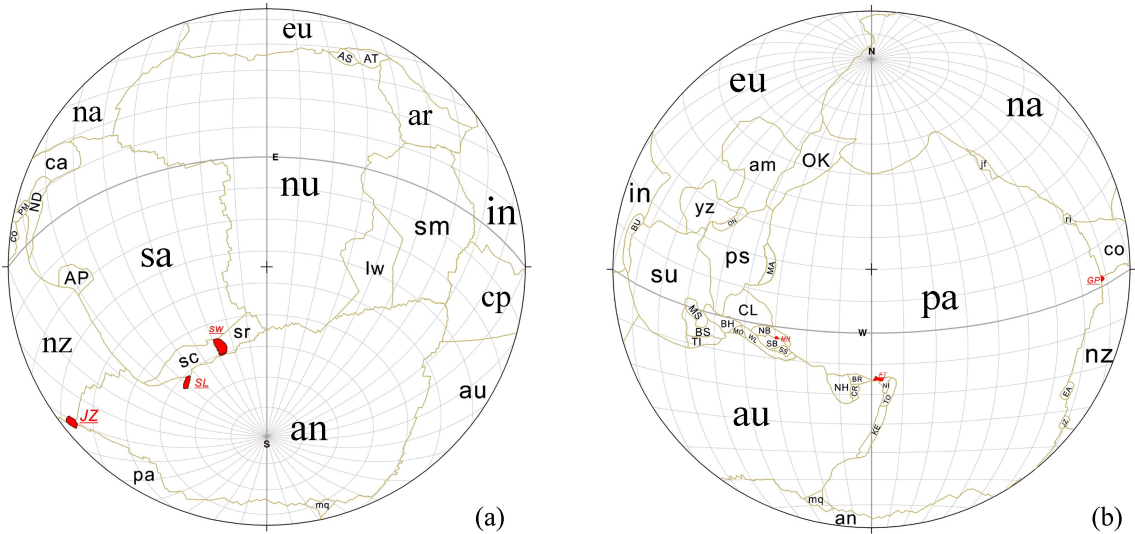


Fig. 2 Plate boundaries and geometries employed for MORVEL
(a) View direction at 35° south, 0° east; (b) View direction at 20° north, 180° east

Table 1 Geometric parameters of 56 modern plates included in the NNR-MORVEL56

Plate ^a	Area ^b	Q_{11}	Q_{22}	Q_{33}	Q_{12}	Q_{13}	Q_{23}
ram	0.130659	0.108248	0.089481	0.063589	0.028732	0.036320	-0.051295
an	1.432624	1.326692	1.174711	0.363845	-0.050954	0.052461	0.081269
AP	0.020501	0.018168	0.004178	0.018656	0.006097	0.002056	-0.005420
ar	0.120824	0.074249	0.066810	0.100589	-0.048782	-0.029553	-0.031041
AS	0.007930	0.003780	0.007017	0.005063	-0.001938	-0.003445	-0.001610
AT	0.014182	0.008022	0.011586	0.008756	-0.003970	-0.005767	-0.003733
au	0.921383	0.597620	0.558686	0.686460	0.223995	-0.217199	0.241071
BH	0.012950	0.007157	0.005807	0.012936	0.006391	-0.000194	0.000201
BR	0.004814	0.000353	0.004786	0.004488	0.000322	-0.001203	0.000086
BS	0.017146	0.011322	0.005961	0.017009	0.007977	-0.000850	0.001173
BU	0.012697	0.012632	0.000436	0.012327	0.000852	0.000130	-0.001936
ca	0.073043	0.066003	0.011971	0.068111	0.018006	-0.005213	0.017059
CL	0.037650	0.015349	0.022487	0.037464	0.018192	0.001581	-0.001323
co	0.072230	0.071072	0.003017	0.070372	-0.005543	0.001064	0.010142
cp	0.203647	0.196537	0.022175	0.188580	-0.021636	0.007182	0.045603
CR	0.003559	0.000414	0.003532	0.003172	0.000289	-0.001100	0.000101
EA	0.004114	0.003554	0.001272	0.003402	-0.001260	-0.000631	-0.001420
eu	1.196311	1.005910	0.894791	0.491921	-0.035559	-0.213221	-0.310262
FT	0.000789	0.000054	0.000787	0.000736	-0.000027	-0.000197	-0.000007
GP	0.000360	0.000346	0.000015	0.000360	-0.000071	0.000002	0.000012
in	0.306360	0.286350	0.042318	0.284052	-0.057049	-0.013096	-0.060490
jf	0.006315	0.005162	0.004356	0.003111	-0.001501	0.001916	0.002491
JZ	0.002406	0.002192	0.000941	0.001679	-0.000560	-0.000394	-0.001032
KE	0.012450	0.003751	0.012432	0.008717	-0.000123	-0.005589	-0.000034
lw	0.117084	0.063137	0.081094	0.089937	-0.043330	0.036053	0.029664
MA	0.010367	0.004018	0.007354	0.009362	0.004369	0.002475	-0.001708
MN	0.000203	0.000050	0.000154	0.000202	0.000086	-0.000011	0.000006

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Table 1 continued from previous page.

Plate ^a	Area ^b	Q_{11}	Q_{22}	Q_{33}	Q_{12}	Q_{13}	Q_{23}
MO	0. 002841	0. 001271	0. 001581	0. 002830	0. 001405	-0. 000126	0. 000113
mq	0. 007890	0. 006131	0. 007510	0. 002139	0. 000812	-0. 003172	0. 001465
MS	0. 010301	0. 007165	0. 003150	0. 010287	0. 004720	-0. 000118	0. 000164
na	1. 365654	1. 228582	0. 941574	0. 561152	0. 066184	-0. 003632	0. 396247
NB	0. 009563	0. 002624	0. 006962	0. 009540	0. 004209	-0. 000360	0. 000211
ND	0. 023942	0. 022362	0. 002000	0. 023523	0. 005755	-0. 000735	0. 002485
NH	0. 015853	0. 001931	0. 015441	0. 014334	0. 002345	-0. 004547	0. 000758
NI	0. 003062	0. 000271	0. 003046	0. 002808	-0. 000212	-0. 000839	-0. 000063
nb	1. 440653	0. 372572	1. 301219	1. 207515	-0. 051346	-0. 005428	0. 044223
nz	0. 396683	0. 385354	0. 068410	0. 339603	-0. 012644	-0. 002969	-0. 113428
OK	0. 074825	0. 053441	0. 066413	0. 029796	0. 012953	0. 030320	-0. 017870
ON	0. 008000	0. 005617	0. 004075	0. 006307	0. 003044	0. 002004	-0. 002552
pa	2. 576858	1. 175689	1. 961254	2. 016772	-0. 429469	0. 077428	-0. 057431
PM	0. 006744	0. 006575	0. 000333	0. 006580	0. 001000	-0. 000159	0. 001021
ps	0. 134118	0. 077744	0. 071266	0. 119226	0. 058301	0. 026648	-0. 027639
ri	0. 002486	0. 002289	0. 000489	0. 002193	-0. 000625	0. 000239	0. 000763
sa	1. 003382	0. 606780	0. 582701	0. 817282	0. 338318	0. 179187	-0. 168608
SB	0. 007615	0. 002101	0. 005575	0. 007554	0. 003341	-0. 000571	0. 000346
sc	0. 041900	0. 036723	0. 034464	0. 012613	0. 005695	0. 011988	-0. 014451
SL	0. 001780	0. 001675	0. 001490	0. 000396	0. 000174	0. 000381	-0. 000634
sm	0. 354795	0. 221034	0. 153743	0. 334814	-0. 154899	0. 024755	0. 035861
sr	0. 027055	0. 018681	0. 026496	0. 008933	0. 001954	0. 012245	-0. 002957
SS	0. 003170	0. 000715	0. 002505	0. 003119	0. 001275	-0. 000352	0. 000183
su	0. 219667	0. 188850	0. 036326	0. 214159	0. 069104	0. 006544	-0. 016832
sw	0. 004543	0. 003525	0. 004269	0. 001292	0. 000527	0. 001817	-0. 000940
TI	0. 008704	0. 005784	0. 003120	0. 008503	0. 004009	-0. 000751	0. 001052
TO	0. 006248	0. 000759	0. 006194	0. 005544	-0. 000536	-0. 001947	-0. 000186
WL	0. 011163	0. 003492	0. 007835	0. 010998	0. 004966	-0. 001074	0. 000660
yz	0. 054249	0. 045687	0. 019960	0. 042851	0. 016644	0. 009648	-0. 019528

^a Plate name abbreviations are as follows; am, Amur; an, Antarctic; AP, Altiplano; ar, Arabia; AS, Aegean Sea; au, Australia; BH, Birds Head; BR, Balmoral Reef; BS, Banda Sea; BU, Burma; ca, Caribbean; CL, Caroline; cp, Cocos; cp, Capricorn; CR, Caroline; EA, Easter; eu, Eurasia; FT, Futuna; GP, Galapagos; in, India; jf, Juan de Fuca; JZ, Juan Fernandez; KE, Kermadec; lw, Lwandle; MA, Mariana; MN, Manus; MO, Maoke; mq, Macquarie; MS, Molucca Sea; na, North America; NB, North Bismarck; ND, North Andes; NH, New Hebrides; NI, Niuafoou; nb, Nubia; nz, Nazca; OK, Okhotsk; ON, Okinawa; pa, Pacific; PM, Panama; ps, Philippine Sea; ri, Rivera; sa, South America; SB, South Bismarck; sc, Scotia; SL, Shetland; sm, Somalia; sr, Sur; SS, Solomon Sea; su, Sundaland; sw, Sandwich; TI, Timor; TO, Tonga; WL, Woodlark; yz, Yangtze. Plate abbreviations given in lower case are for plates included in the MORVEL. Plate abbreviations given in the upper case are for plates from Bird (2003); b. Plate areas are in steradians for a unit sphere, and the sum of which totals 4π .

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The FORTRAN90 Program

MODULE CALCULATE_SPHPOLAERA_INERTEN

```
!
! THIS IS A FORTRAN90 MODULE, WHICH INCLUDES A SET OF FUNCTIONS AND SUBROUTINES
! USED FOR CALCULATING THE AREAS AND INERTIA TENSORS OF AMOUNTS SPHERICAL POLYGONS.
! * * * * *
! DISCUSSION:
! VERTEXES OF THE SPHERICAL POLYGON MUST BE SEQUENCED IN A COUNTERCLOCKWISE DIRECTION.
! THE FIRST POINT IN BOUNDARY FILES SHOULD ALWAYS BE IDENTICAL TO THE LAST ONE
! IN ORDER TO CONSTRUCT A CLOSED POLYGON OF N SIDES WITH N+1 DATA POINTS.
!
! MODIFIED: 26 DECEMBER 2014
!
! AUTHOR: CHUNXIAO LI
!
!
! IMPLICIT NONE
REAL * 8, PARAMETER: PI = 3.1415926535897932D0
```

```

REAL * 8, PARAMETER:: HALFPI = PI/2D0
REAL * 8, PARAMETER:: DEGREE = 180D0/PI!DEGREES PER RADIAN
CONTAINS
! -----
! SPITC RETURNS THE AREA AND SIX COMPONENTS OF THE INERTIA TENSOR OF A N-SIDED
! SPHERICAL POLYGON.
! THE LAT AND LON SHOULDE TAKE DEGREE AS THEIR UNIT, RANGING FROM -90 TO 90 AND
! FROM -180 TO 180, RESPECTIVELY.
! THE N+1 DATA POINTS PORTRAY A POLYGON WIYH N VERTEXES.
! THE OUTPUT PARAMETER, SPHERICALPOLYGONAREA, REPRESENTS THE AREA OF THE SPHERICAL POLYGON
! IN STERADIANS FOR A UNIT SPHERE.
! THE REMAMENT SIX PARANETERS INDICATE SIX COMPONENTS OF THE INERTIA TENSOR
SUBROUTINE SPITC(LAT,LON,N,SPHERICALPOLYGONAREA,SUM11,SUM22,SUM33,SUM12,SUM13,SUM23)
    IMPLICIT NONE
    INTEGER:: N
    REAL * 8:: LAT(N),LON(N)
    REAL * 8:: SPHERICALPOLYGONAREA
    REAL * 8:: SUM11,SUM22,SUM33,SUM12,SUM13,SUM23
    INTEGER:: J
    REAL * 8:: LON1,LON2,LAT1,LAT2
    REAL * 8:: HAVB,DLON,PDLON
    REAL * 8:: T,A,B,C,S,SUM,EXCESS
    REAL * 8:: C1,C2,C3
    COMMON/GROUP1/ C1,C2,C3
    REAL * 8:: S11,S22,S33,S12,S13,S23
    REAL * 8:: EPS
    COMMON/GROUP2/EPS!THE TOLERANCE INVOLVED ON COMPUTATION IS SPECIFIED BY EPS.
    SUM = 0D0
    SUM11 = 0D0
    SUM22 = 0D0
    SUM33 = 0D0
    SUM12 = 0D0
    SUM13 = 0D0
    SUM23 = 0D0
    DOJ = 1,N-1
        LON1 = LON(J)/DEGREE
        LAT1 = LAT(J)/DEGREE
        LON2 = LON(J+1)/DEGREE
        LAT2 = LAT(J+1)/DEGREE
        CALL COEFFICIENT(LAT1,LON1,LAT2,LON2)
        PDLON = LON2-LON1
        DLON = ABS(PDLON)
        IF(DLON.GT.1E-6) THEN
            IF(DLON.GT.PI) DLON = 2D0 * PI-DLON
            IF(LON2.LT.LON1.AND.PDLON.LT.-PI) LON2 = LON2 + 2D0 * PI
            IF(LON2.GT.LON1.AND.PDLON.GT.PI) LON2 = LON2 -2D0 * PI
            HAVB = HAV(LAT2-LAT1)+COS(LAT1) * COS(LAT2) * HAV(DLON)
            B = 2D0 * ASIN(SQRT(HAVB))
            A = HALFPI-LAT1
            C = HALFPI-LAT2
            S = 0.5D0 * (A+B+C)
            T = TAN(S/2D0) * TAN((S-A)/2D0) * TAN((S-B)/2D0) * TAN((S-C)/2D0)
            EXCESS = ABS(4D0 * ATAN(SQRT(ABS(T))))
            IF(LON2.LT.LON1) EXCESS = -EXCESS
            SUM = SUM + EXCESS
            CALL FSIM2(LON1,LON2,FS_LOWERLIMIT,FS_UPPERLIMIT,F11,EPS,S11)
            CALL FSIM2(LON1,LON2,FS_LOWERLIMIT,FS_UPPERLIMIT,F22,EPS,S22)
            CALL FSIM2(LON1,LON2,FS_LOWERLIMIT,FS_UPPERLIMIT,F33,EPS,S33)
            CALL FSIM2(LON1,LON2,FS_LOWERLIMIT,FS_UPPERLIMIT,F12,EPS,S12)
            CALL FSIM2(LON1,LON2,FS_LOWERLIMIT,FS_UPPERLIMIT,F13,EPS,S13)
            CALL FSIM2(LON1,LON2,FS_LOWERLIMIT,FS_UPPERLIMIT,F23,EPS,S23)
            SUM11 = SUM11 + S11
            SUM22 = SUM22 + S22
            SUM33 = SUM33 + S33
            SUM12 = SUM12 + S12
            SUM13 = SUM13 + S13
    ENDDO

```

```

        SUM23 = SUM23 + S23
    END IF
END DO
IF(SUM.LT.0.D0) THEN
    SPHERICALPOLYGONAREA = SUM + 4D0 * PI
    SUM11 = 8.D0/3.D0 * PI-SUM11-ABS(SUM)
    SUM22 = 8.D0/3.D0 * PI-SUM22-ABS(SUM)
    SUM33 = 8.D0/3.D0 * PI-SUM33-ABS(SUM)
ELSE
    SPHERICALPOLYGONAREA = SUM
    SUM11 = SPHERICALPOLYGONAREA - SUM11
    SUM22 = SPHERICALPOLYGONAREA - SUM22
    SUM33 = SPHERICALPOLYGONAREA - SUM33
END IF
SUM12 = -SUM12
SUM13 = -SUM13
SUM23 = -SUM23
RETURN
END SUBROUTINE
! -----
! GIVEN THE DOMAIN OF INTEGRAL, FSIM2 RETUENS THE INTEGRAL OF F.
SUBROUTINE FSIM2(A,B,FS_LOWERLIMIT,FS_UPPERLIMIT,F,EPS,S)
    IMPLICIT NONE
    REAL * 8,EXTERNAL :: FS_LOWERLIMIT,FS_UPPERLIMIT,F
    REAL * 8 :: A,B,EPS,S
    ! LOCAL VARIABLES
    REAL * 8 :: H,S1,S2,TS1,TS2,X,G,S0,C
    INTEGER :: N,J
    N=1
    H=0.5D0 * (B-A)
    C=(B-A) * 1.0E-06
    CALL SIMP1(A,FS_LOWERLIMIT,FS_UPPERLIMIT,F,EPS,S1)
    CALL SIMP1(B,FS_LOWERLIMIT,FS_UPPERLIMIT,F,EPS,S2)
    TS1=H * (S1+S2)
    DO WHILE (ABS(H).GE.ABS(C))
        X=A-H
        TS2=0.5D0 * TS1
        DO J=1,N
            X=X+2D0 * H
            CALL SIMP1(X,FS_LOWERLIMIT,FS_UPPERLIMIT,F,EPS,G)
            TS2=TS2+H * G
        END DO
        S=(4D0 * TS2-TS1)/3.0D0
        N=N+N
        IF (N.GE.16) THEN
            IF (ABS(S-S0).LE.EPS * (ABS(S)+1.0D0)) RETURN
        END IF
        S0=S
        TS1=TS2
        H=0.5D0 * H
    END DO
    RETURN
END SUBROUTINE

SUBROUTINE SIMP1(X,FS_LOWERLIMIT,FS_UPPERLIMIT,F,EPS,G)
    IMPLICIT NONE
    REAL * 8 :: X,EPS,G
    REAL * 8,EXTERNAL :: FS_LOWERLIMIT,FS_UPPERLIMIT,F
    ! LOCAL VARIABLES
    REAL * 8 :: Y1,Y2,H,C,TS1,TS2,Y,G0
    INTEGER :: N,I
    N=1
    Y1 = FS_LOWERLIMIT(X)
    Y2 = FS_UPPERLIMIT(X)
    H=0.5D0 * (Y2-Y1)
    C=(Y2-Y1) * 1.0E-06

```

```

      TS1=H*(F(X,Y1)+F(X,Y2))
      DO WHILE (ABS(H).GE.ABS(C))
        Y=Y1-H
        TS2=0.5D0*TS1
        DO I=1,N
          Y=Y+2D0*H
          TS2=TS2+H*F(X,Y)
        END DO
        G=(4D0*TS2-TS1)/3.0D0
        N=N+N
        IF (N.GE.16) THEN
          IF (ABS(G-G0).LE.EPS*(ABS(G)+1.0D0)) RETURN
        END IF
        G0=G
        TS1=TS2
        H=0.5D0*H
      END DO
      RETURN
    END SUBROUTINE

```

!-----

! FS_LOWERLIMIT RETURN THE LOWERLIMIT CONDUCTING AS A FUNCTION OF
! LONGITUDE; AND THE FS_UPPERLIMIT RETURNS A CONSTANT UPPERLIMIT, PI/2.

```

FUNCTION FS_LOWERLIMIT(LON)
  IMPLICIT NONE
  REAL * 8 :: FS_LOWERLIMIT
  REAL * 8 :: LON
  REAL * 8 :: C1,C2,C3
  COMMON /GROUP1/ C1,C2,C3
  FS_LOWERLIMIT=-ATAN((C1*COS(LON)+C2*SIN(LON))/C3)
  RETURN
END FUNCTION

```

```

FUNCTION FS_UPPERLIMIT(LON)
  IMPLICIT NONE
  REAL * 8 :: FS_UPPERLIMIT
  REAL * 8 :: LON
  FS_UPPERLIMIT=HALFPI
  RETURN
END FUNCTION

```

! COEFFICIENT RETURNS THREE COEFFICIENTS RELATING THE SPHERICAL COORDINATES OF ANY
! TWO ADJACENT VERTEXES OF THE POLYGON.

```

SUBROUTINE COEFFICIENT(LAT1,LON1,LAT2,LON2)
  IMPLICIT NONE
  REAL * 8 :: LAT1,LON1,LAT2,LON2
  REAL * 8 :: C1,C2,C3
  COMMON /GROUP1/ C1,C2,C3
  C1 = COS(LAT1)*SIN(LON1)*SIN(LAT2)-COS(LAT2)*SIN(LON2)*SIN(LAT1)
  C2 = COS(LAT2)*COS(LON2)*SIN(LAT1)-COS(LAT1)*COS(LON1)*SIN(LAT2)
  C3 = COS(LAT1)*COS(LAT2)*SIN(LON2-LON1)
  RETURN
END SUBROUTINE

```

!-----
! THE FOLLOWING INTEGRANDS CORRESPOND SIX COMPONENTS OF THE INERTIA TENSOR.

```

FUNCTION F11(LON,LAT)
  IMPLICIT NONE
  REAL * 8 :: F11
  REAL * 8 :: LAT,LON
  F11 = COS(LAT)**3*COS(LON)**2
  RETURN
END FUNCTION

```

```

FUNCTION F22(LON,LAT)
  IMPLICIT NONE
  REAL * 8 :: F22
  REAL * 8 :: LAT,LON
  F22 = COS(LAT)**3*SIN(LON)**2

```

```

      RETURN
END FUNCTION

FUNCTION F33(LON,LAT)
  IMPLICIT NONE
  REAL * 8 :: F33
  REAL * 8 :: LAT,LON
  F33 = SIN(LAT) * * 2 * COS(LAT)
  RETURN
END FUNCTION

FUNCTION F12(LON,LAT)
  IMPLICIT NONE
  REAL * 8 :: F12
  REAL * 8 :: LAT,LON
  F12 = COS(LAT) * * 3 * COS(LON) * SIN(LON)
  RETURN
END FUNCTION

FUNCTION F13(LON,LAT)
  IMPLICIT NONE
  REAL * 8 :: F13
  REAL * 8 :: LAT,LON
  F13 = COS(LAT) * * 2 * SIN(LAT) * COS(LON)
  RETURN
END FUNCTION

FUNCTION F23(LON,LAT)
  IMPLICIT NONE
  REAL * 8 :: F23
  STHZREAL * 8 :: LAT,LON
  F23 = COS(LAT) * * 2 * SIN(LAT) * SIN(LON)
  RETURN
END FUNCTION
! -----
! HAVERSINE FUNCTION
FUNCTION HAV(X)
  IMPLICIT NONE
  REAL * 8 :: X
  REAL * 8 :: HAV
  HAV = (1D0-COS(X))/2D0
  RETURN
END FUNCTION
! * * * * *
END MODULE

```

MORVEL 构造板块的转动张量

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摘要：NNR-MORVEL56 板块运动模型描述了全球 56 个构造板块在无整体旋转参考架下的角速度运动参数。这些板块可以近似描述为单位球上的无重叠球面多边形区域。用 ITRF 速度场计算这 56 个板块相对于无整体旋转参考架下的绝对运动时，板块的几何参数起着至关重要的作用。详细给出了计算板块几何参数的方法并且编写了 FORTRAN90 程序以供参考，使得计算单位球上板块的面积和转动惯性张量得以实现。文中的计算方法和程序主要采用球面三角算法和自适应辛普森双积分算法，并对全球 56 个板块的几何参数进行了计算，得到了较为可靠的计算结果。

关键词：构造板块；球面多边形；转动惯量张量；NNR-MORVEL56

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